San Diego Math League Middle School Division, Round 1a November 16, 2013

1.	The population of Facepalm decreased by 20% from 2003 to 2008. By 2013, the population returned to
	its 2003 value. By what percent did the population of Facepalm increase between 2008 and 2013?

(A) $16\frac{2}{3}$

(B) 20

(C) 25

(D) 30

(E) 32

2. A positive integer n is frabjous if the sum of the digits of n! is divisible by n. Which of the following numbers is frabjous?

(A) 5

(B) 6

(C)7

(D) 8

(E) 9

3. How many distinct prime factors does $4^8 - 81$ have?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

4. In $\triangle ABC$, $\angle A = 2\angle B = 6\angle C$. What is the measure of $\angle A$?

(A) 20°

(B) 32°

(C) 90°

(D) 108°

(E) 120°

5. Of the 800 students that attend Mebane Middle School, 3/5 take a gym class, and 1/4 take an art class. If 3/10 of the students take neither gym nor art, then how many students take both gym and art?

(A) 75

(B) 80

(C) 120

(D) 140

(E) 180

6. How many distinct scalene triangles have integer side lengths and a perimeter of 22?

(A) 5

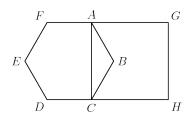
(B) 6

(C) 8

(D) 10

(E) 11

7. Regular hexagon ABCDEF and square AGHC are shown below. If the length of segment AF is 4, then what is the area of pentagon AGHCB?



(A) $32 - 4\sqrt{3}$

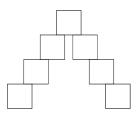
(B) $32 - 2\sqrt{3}$

(C) $48 - 8\sqrt{3}$

(D) $48 - 4\sqrt{3}$

(E) 48

8. Bradley glues seven identical cubes together to form a stack, the front view of which is shown below. With the exception of the top cube, 1/3 of the top face of each cube is covered by the cube above it. Bradley dips his stack of cubes in red paint. What is the ratio of the surface area of the stack that is painted red to the surface area of top cube that is painted red?



(A) 57:8

(B) 15:2

(C) 34:5

(D) 20:3

(E) 6:1

9. The median of the first six numbers in an arithmetic sequence is 20. The median of the first nine numbers in the same sequence is 44. What is the first number in the sequence?

(A) -32

(B) -20

(C) -9

(D) 0

(E) 3

10. What is the angle formed by the minute and hour hands of a standard 12-hour clock when the time is 2:55?

(A) 87.5°

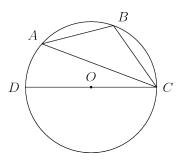
(B) 92.5°

(C) 105°

(D) 117.5°

(E) 120°

11. In the diagram below, O is the center of the circle. If $\angle BAC = 35^{\circ}$, find $\angle BCD$.



(A) 35°

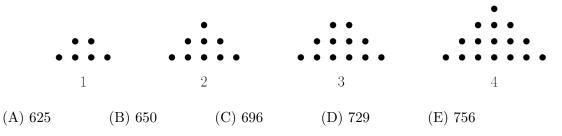
(B) 40°

(C) 45°

(D) 50°

(E) 55°

12. Annie is drawing a pattern of dot pyramids. The first four dot pyramids are shown below. If the pattern continues, how many dots will be in Annie's 51st dot pyramid?



13. License plates in Barnia consist of four characters. Each character is either a digit (0-9), or a letter (A, B, or C). The same character may not appear more than once on a plate. How many unique license plates can be made that do not have three numerical digits in a row?

(A) 4320

(B) 4920

(C) 7800

(D) 7860

(E) 12120

14. Ben can eat a gallon of ice cream in 2 hours. His friend Jerry can eat 2 gallons of ice cream in 3 hours. Ben starts eating from a 2 1/2 gallon vat of ice cream. Later, Jerry joins him and the two eat together until all the ice cream is gone. If it takes 3 hours from when Ben starts eating to when the ice cream is finished, then how many gallons does Ben eat before Jerry arrives?

(A) 3/5

(B) 3/4

(C) 5/6

(D) 7/12

(E) 3/2

15. Rocky, Adrian, and Mickey take turns flipping a coin. Rocky flips first, followed by Adrian, followed by Mickey. What is the probability that Rocky is the first person whose coin lands on heads?

(A) 1/2

(B) 4/7

(C) 5/7

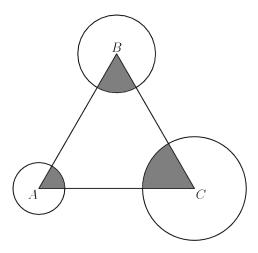
(D) 7/13

(E) 7/11

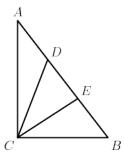
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- 1. C
- 2. E
- 3. C
- 4. D
- 5. C
- 6. A
- 7. D
- 8. A
- 9. B
- 10. D
- 11. E
- 12. E
- 13. C
- 14. B
- 15. B

- 1. What is the units digit of 2013^{2013} ?
- 2. Niki has 15 dollars more than twice as much money as her sister Amy. If Niki gives Amy 30 dollars, then Niki will have half as much money as her sister. How many dollars does Niki have?
- 3. Simplify $\sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$.
- 4. Equilateral triangle ABC has side length 6. Circles with centers at A, B, and C are drawn such that their respective radii r_A , r_B , and r_C form an arithmetic sequence with $r_A < r_B < r_C$. If the shortest distance between circles A and B is 3.5, and the shortest distance between circles A and C is 3, then what is the area of the shaded region? Express your answer in terms of pi.



- 5. The probability of drawing a red marble from a bag is 3/5. After some red marbles are removed, the probability of drawing a red marble is 2/7. What is the smallest number of marbles that could have originally been in the bag?
- 6. The base 5 number 32 is equal to the base 7 number 23. There are two 3-digit numbers in base 5 which similarly have their digits reversed when expressed in base 7. What is their sum, in base 5? (You do not need to include the base 5 subscript in your answer).
- 7. In the right triangle ABC shown, E and D are the trisection points of the hypotenuse AB. If CD = 7 and CE = 6, what is the length of hypotenuse AB? Express your answer in simplest radical form.



8. On a windless day, a pigeon can fly from Albatrocity to Finchester and back in 3 hours and 45 minutes. However, when there is a 10 mile per hour wind blowing from Albatrocity to Finchester, it takes the pigeon 4 hours to make the round trip. How many miles is it from Albatrocity to Finchester?

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- 1. 3
- 2. 55

- 3. 34. $\frac{29\pi}{24}$ 5. 256. 11037. $3\sqrt{17}$ 8. 75